# **TRI-BAND RAT-RACE COUPLER USING RESONATORS**

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This paper presents a novel method to design a tri-band rat-race coupler by using resonators in  $\pi$ -shaped structure. The proposed structure demonstrates tri-band performance and a compact size due to  $\pi$ -shaped structure. In order to achieve tri-band operation, we use resonators and stubs with conventional rat-race coupler structure. By sharing the stub with resonator of two adjacent  $\pi$ -shaped circuits and making stubs inside the tri-band rat-race coupler, compactness is well kept. Compare to the size of 1 GHz conventional rat-race coupler, 84% reduction is archived by our proposed structure. We demonstrate that insertion losses are better than 4 dB, the reflection coefficients are better than 10 dB, and the isolations are better than 22 dB. In phase difference is less than 2.3°.

Index Terms—resonator, rat-race coupler, tri-band,  $\pi$  shaped structure.

#### I. INTRODUCTION

Interoperability and co-existence between multi standards is the main critical issue in modern communication systems [1]. To satisfy this harsh requirement, multi-band or broadband radio systems are proposed to be used at different frequencies associated with different standards. To be used in multiband or broadband radio system, it is required to develop new type of multiband component such as power combiner, hybrid coupler and rat-race coupler. Also one of the requirements of developing the various microwave components is to reduce the size, complexity and cost [2]. Especially in this paper, we will present a novel type of multiband rat-race coupler.

Rat-race coupler is widely used in multi-standard microwave system. For example, in class-D switching-mode power amplifiers [3], and in the transmitter end of MIMO systems [4], it is widely used for dividing an input signal into two signals having 0° and 180° phase difference [5]. However, due to  $\lambda/4$  transmission line, traditional design is limited at one single band [5]-[7]. Recently, the following researches are presented to realize multi-band operation of rat-race coupler. Dual-band quarter-wave composite right/left-handed transmission line (CRLH TL) is presented [5], and applied in rat-race coupler [4], [5], [8]. Stepped-impedance microstrip line is analyzed and used for dual-band operation [2], [6]. The C-section together with two transmission line sections is proposed and synthesized [1]. Size reduction is considered in design procedure [2], [4], [9].

In this paper, we present a novel compact tri-band rat-race coupler by using resonators and open stubs at both ends of  $\lambda/4$  transmission line.

## II. PROPOSED TRI-BAND TOPOLOGY

The conventional rat-race coupler is shown in Fig. 1.

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Fig. 1. Conventional rat-race coupler structure.

The proposed topology of tri-band rat-race coupler is shown in Fig. 2. The characteristic impedance is  $Z_0=50 \Omega$ .



Fig. 2. The topology of the proposed tri-band rat-race coupler.

To make a tri-band rat-race coupler, the resonator is used in  $\pi$  -shaped structure as shown in Fig.3. Resonance is created by capacitor and inductor in parallel. The relationship is shown in (1).

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

Before analyzing the whole topology of tri-band rat-race coupler, we will explain  $\pi$ -shaped structure with resonators in Fig. 3.

The line  $l_1$  is  $\lambda/4$  length at frequency  $f_1$  with the impedance of  $Z_1$ =70.7  $\Omega$ . At both ends of  $l_1$ , we put one stub with impedance  $Z_2$  with length  $l_2$  between resonator  $f_1$  and resonator  $f_2$ . After resonator  $f_2$ , we add one open stub with impedance  $Z_2$  and length  $l_3$ . Resonators  $f_1$  and  $f_2$  are specially designed to block the signal generated at frequency  $f_1$  and  $f_2$ , respectively.



Fig. 3. The proposed  $\pi$ -shaped structure.

Fig. 4 shows the principle of  $\pi$ -shaped structure. At frequency  $f_1$ , the signal is blocked by the first resonator  $f_1$  and the proposed  $\pi$ -shaped structure becomes a simple transmission line (Fig. 4(a)). At frequency  $f_2$ , the signal passes through resonator  $f_1$  and be blocked by resonator  $f_2$ . Our proposed  $\pi$ -shaped structure transforms to a simple  $\pi$ -shaped circuit as shown in Fig. 4(b). Finally at frequency  $f_3$ , the first stub  $l_2$  is added to the second stub  $l_3$  to be considered as one longer open stub  $l_2$ + $l_3$  (Fig. 4(c)).



Fig. 4. Principle of tri-band operation.

In order to calculate the length  $l_2$  and  $l_2+l_3$  and impedance  $Z_2$ , we use ABCD matrix that allows simplifying the calculation.

At  $f_2$ , Fig. 4(b) with symmetrical structure at both ends of

l<sub>1</sub>corresponds to the following matrix [3]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY_2 \tan \beta_2 l_2 & 1 \end{bmatrix} * \begin{bmatrix} \cos \beta_2 l_1 & jZ_1 \sin \beta_2 l_1 \\ jY_1 \sin \beta_2 l_1 & \cos \beta_2 l_1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ jY_2 \tan \beta_2 l_2 & 1 \end{bmatrix}$$
(2)

Where 
$$\beta_2 = \frac{2\pi}{\lambda_2}, l_1 = \frac{\lambda_1}{4}, Z_1 = 70.7\Omega$$

At frequency  $f_2$ , ABCD matrix of a quarter wavelength  $(\lambda/4)$  transmission line with impedance  $Z_{01}$  can be expressed as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jZ_{01} \\ jY_{01} & 0 \end{bmatrix}$$
(3)

Similarly, at  $f_3$ , the matrix of Fig. 4(c) will be:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY_2 \tan \beta_3(l_2 + l_3) & 1 \end{bmatrix} * \begin{bmatrix} \cos \beta_3 l_1 & jZ_1 \sin \beta_3 l_1 \\ jY_1 \sin \beta_3 l_1 & \cos \beta_3 l_1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ jY_2 \tan \beta_3(l_2 + l_3) & 1 \end{bmatrix}$$
(4)
Where  $\beta_3 = \frac{2\pi}{\lambda_3}, l_1 = \frac{\lambda_1}{4}, Z_1 = 70.7\Omega.$ 

The quarter wavelength transmission line with impedance  $Z_{02}$  at frequency  $f_3$  can be expressed as:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 0 & jZ_{02} \\ jY_{02} & 0 \end{bmatrix}$$
(5)

Ideally the equivalent length of the proposed  $\pi$ -shaped circuit should be  $\lambda_1 / 4$ ,  $\lambda_2 / 4$  and  $\lambda_3 / 4$  at frequency  $f_1$ ,  $f_2$ , and  $f_3$ , respectively. To do that, we have to satisfy the condition of rat-race coupler for these three frequencies.

By equalizing the equations (2) and (3), we can deduce:

$$A = \cos \beta_2 l_1 - Y_2 Z_1 \tan \beta_2 l_2 \sin \beta_2 l_1 = 0$$
  
$$\Rightarrow Z_1 \tan \beta_2 l_1 = Z_2 \cot \beta_2 l_2$$
(6)

$$B = jZ_1 \sin \beta_2 l_1 = jZ_{01}$$
  
$$\Rightarrow Z_{01} = Z_1 \sin \beta_2 l_1$$
(7)

$$C = jY_2 \tan \beta_2 l_2 \cos \beta_2 l_1 + jY_1 \sin \beta_2 l_1 - jY_2 Y_2 Z_1 \tan \beta_2 l_2$$
  
\* \tan \beta\_2 l\_2 \sin \beta\_2 l\_1 + jY\_2 \tan \beta\_2 l\_2 \cos \beta\_2 l\_1 = jY\_{01} (8)

$$D = \cos \beta_2 l_1 - Y_2 Z_1 \tan \beta_2 l_2 \sin \beta_2 l_1 = 0$$
  

$$\Rightarrow Z_1 \tan \beta_2 l_1 = Z_2 \cot \beta_2 l_2$$
(9)

Similarly for frequency  $f_3$ , we can deduce:

$$\Rightarrow Z_1 \tan \beta_3 l_1 = Z_2 \cot \beta_3 (l_2 + l_3) \tag{10}$$

$$\Rightarrow Z_{02} = Z_1 \sin \beta_3 l_1 \tag{11}$$

Where 
$$\beta_2 = \frac{2\pi}{\lambda_2}, \beta_3 = \frac{2\pi}{\lambda_3}, l_1 = \frac{\lambda_1}{4}, Z_1 = 70.7\Omega.$$

In equation (9), by giving  $Z_2=50 \Omega$ , we can calculate the stub length  $l_2$ . By (7),  $Z_{01}$  is obtained as  $57.2 \Omega$ . With the same way using (10) we can find the value of  $l_2+l_3$  and  $Z_{02} = 41.6 \Omega$ . The calculated  $Z_{01}$  and  $Z_{02}$  are far from the ideal value 70.7  $\Omega$ . We have to increase the value of  $Z_1$  in (7) and (11) to obtain the best performance of tri-band operation.

## III. SIMULATED AND MEASURED RESULTS



Fig. 5. Fabricated tri-band rat-race coupler.

We fabricate circuit by using the substrate Taconic TLX-8 with dielectric constant 2.55 at the frequencies 1GHz/1.5GHz/2.5GHz as shown in Fig. 5.



Fig. 6. Simulated and measured results of insertion loss of  $S_{23}$  and  $S_{43}$ .

By combining two adjacent  $\pi$ -shaped lines as one, the number of these lines added on initial rat-race coupler is reduced to six instead of twelve. Because our proposed

structure is based on the conventional rat-race coupler for 2.5 GHz, the size of our coupler is 84% smaller than conventional rat-race coupler at 1 GHz. The measured and simulated results are compared in Fig. 6 to Fig. 12. For three frequencies, insertion losses are better than 4 dB. The reflection coefficients are better than 10 dB, and the isolation between port 1 and 3, port 2 and 4 are better than 26 dB and 22dB, respectively. In phase difference  $\angle$  S23- $\angle$  S43 is less than 2.3°. Out-of-phase difference  $\angle$  S21- $\angle$ S41 is less than 183.8°.



Fig. 7. Simulated and measured results of  $\angle S_{23}$ - $\angle S_{43}$ .



Fig. 8. Simulated and measured results of insertion loss of  $S_{21}$  and  $S_{41}$ .



Fig. 9. Simulated and measured results of  $\angle S_{21}$ - $\angle S_{41}$ .



Fig. 10. Simulated and measured results of reflection coefficient of  $S_{11}$  and  $S_{44}$ .



Fig. 11. Simulated and measured results of reflection coefficient of  $S_{22}$  and  $S_{33}$ .



Fig. 12. Simulated and measured results of isolation of S<sub>31</sub> and S<sub>42.</sub>

## IV. CONCLUSION

A simple and compact tri-band rat-race coupler at 1 GHz,

1.5 GHz and 2.5 GHz frequency has been presented in this paper. We use  $\pi$ -shaped structure which contains four resonators and microstrip lines. By using  $\pi$ -shaped circuits with resonators, one compact tri-band coupler is obtained. In addition, because we designed our tri-band coupler based on the highest frequency at 2.5 GHz, compare to 1GHz conventional rat-race coupler, the size of our tri-band rat-race coupler is 84% smaller. The measured result of our proposed rat-race coupler is comparable with the previous work for dual-band operation even though our coupler is working for tri-band application [6]. This methodology can be used further for multiband application.

## REFERENCES

- Yi-Chyun Chiou, Jen-Tsai Kuo, and Chi-Hung Chan, "New miniaturized dual-band rat-race coupler with microwave C-section," *IEEE MTT-S International Microwave Symposium Digest*, pp. 701-704, June 2009.
- [2] Ching-Luh Hsu, Chin-Wei Chang, and Jen-Tsai Kuo, "Design of dualband microstrip rat-race coupler with circuit miniaturization," *IEEE MTT-S International Microwave Symposium*, pp. 177-180, June 2007.
- [3] Pouya Aflaki, Renato Negra, and Fadhel M. Ghannouchi, "Dual-band rat-race balun structure using transmission-lines and lumped component resonators," *IEEE MTT-S International Microwave Symposium Digest*, pp. 1572-1575, May 2010.
- [4] Pei-Ling Chi, Cheng-Jung Lee, and Tatsuo Itoh, "A compact dual-band metamaterial-based rat-race coupler for a MIMO system application," *IEEE MTT-S International Microwave Symposium Digest*, pp. 667-670, June 2008.
- [5] I-Hsiang Lin, Marc DeVincentis, Christophe Caloz, and Tatsuo Itoh, "Arbitrary dual-band components using composite right/left-handed transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, vol. 52, no. 4, pp. 1142-1149, April 2004.
- [6] Kuo-Sheng Chin, Ken-Min Lin, Yen-Hsiu Wei, Tzu-Hao Tseng, and Yu-Jie Yang, "Compact dual-band branch-line and rat-race couplers with stepped-impedance-stub lines," *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 5, pp. 1213-1221, May 2010.
- [7] C. P. Kong, and Kwok-Keung M. Cheng, "Dual-band rat-race coupler with bandwidth enhancement," *IEEE MTT-S International Microwave Symposium Digest*, pp. 1559-1562, June 2006.
- [8] Yuandan Dong and Tatsuo Itoh, "Application of composite right/lefthanded half-mode substrate integrated waveguide to the design of a dual-band rat-race coupler," *IEEE MTT-S International Microwave Symposium Digest*, pp. 712-715, May 2010.
- [9] Gerard Siso, Jordi Bonache, and Ferran Martin, "Dual-band rat-race hybrid coupler implemented through artificial lines based on complementary split ring resonators," *IEEE MTT-S International Microwave Symposium Digest*, pp. 625-628, June 2009.